Topics for the Final Exam

- Definitions and basic properties of groups, rings, and fields.

- Subgroups.

- Cyclic groups.

- Dihedral groups D_n.

- Permutation groups (including the symmetric and alternating groups, S_n and A_n).

- The subgroups $Z(G)$, $C_G(a)$, $C_G(H)$, and $N_G(H)$.

- Normal subgroups, cosets, and quotient (aka factor) groups.

- Lagrange's Theorem.

- The product HK of two subgroups.

- Group homomorphisms and isomorphisms, including the First Isomorphism Theorem.

- Subrings and ideals.

- Quotient rings.

- Maximal and prime ideals.

- Integral domains.

- Polynomial rings.

- Factorization of polynomials, including irreducibility criteria.

- Irreducible and prime elements of an integral domain.

- Unique Factorization Domains, Principal Ideal Domains, and Euclidean Domains.

- Construction of finite fields.

- Extension fields.

- Algebraic extensions, including minimal polynomials and the degree of an extension.

These topics are covered in chapters 1 through 7, 9, 10, 12, 13, 14, 16, 17, 18, 20, and 21 of your textbook.